Closing today: HW_4A, 4B, 4C (6.4,6.5)
Closing Wed: HW_5A, 5B, 5C (7.1,7.2,7.3)

7.1 Integration by Parts

Goal: We will reverse the product rule.
This method will help us evaluate integrals involving products, logs or inverse trig.

Before we start, add these to your basic list of integrals:

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + C$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + C$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + C$$

Derivation of Integration By Parts

The product rule says:

$$u(x)v'(x) + v(x)u'(x) = \frac{d}{dx}(u(x)v(x))$$

which can be written as

$$\int u(x)v'(x)dx + \int v(x)u'(x)dx = u(x)v(x)$$

Writing this in terms of the differentials:

$$dv = v'(x)dx \text{ and } du = u'(x)dx$$

we have

$$\int u\,dv + \int v\,du = uv$$

which we rearrange to get

Integration by Parts formula:

$$\int u\,dv = uv - \int v\,du$$

Example:

$$\int x\cos(x)dx$$

Step 1: Choose *u* and *dv*.
Step 2: Compute *du* and *v*.
Step 3: Use formula (and hope)

Notes:

1. The symbols *u* and *v* don't ever appear in the integration. They are just locations in the formula (no variables are changing, this is not substitution).

2. *u* and *dv* completely split up the integrand. So once you chose *u*, then *dv* is everything else.

3. The goal is to make $\int v\, du$ "nicer" than $\int u\, dv$
   (a) Pick u = "something that gives a derivative that is simpler than the original u"
   (b) Pick dv = "something that you can integrate"
   (c) And see if v du is something in our table!